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Asymptotic Eigenequations and Analytic Formulas for the Dispersion Characteristics of Open Wide Microstrip Lines

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Abstract—**Through the matched asymptotic expansions technique, asymptotic eigenequations for the even and odd modes of an open wide microstrip transmission line are derived. The eigenequations, and simplifications thereof which do not involve integration, can be solved easily for the effective permittivity. Even though d/W is assumed to be small, the solutions are good even if $d/W \simeq 0.8$ when compared with the numerical results of Jansen [19]. From these eigenequations, asymptotic formulas for the effective permittivity can be derived which are excellent when $d/W \simeq 0.2$. When the frequency goes to zero, we reproduced the asymptotic formula derived under the quasi-TEM approximation in [8]. The asymptotic analysis provides good physical insight into the problem, otherwise unavailable from numerical analysis.**

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I. Introduction

EVER SINCE the introduction of microstrip transmission lines, the field of microwave engineering has been inundated with papers on the calculation of the characteristic impedance and effective permittivity of a microstrip transmission line. Due to the increasing use of microstrip lines in the high-frequency regime, numerous papers on the study of the dispersion characteristics and higher order modes of the line have been published. Excel-

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lent summaries and reviews of the methods employed in the studies are given by Mittra and Itoh [1] and Kuester and Chang [2]–[4].

Analytic formulas for the quasi-TEM dispersionless characteristic impedance of a microstrip line separated by a dielectric slab was first derived by Wheeler [5], [6] and recently by Poh *et al.* [7]. Mittra and Itoh [8] have also employed the semianalytic generalized Wiener–Hopf technique to study the quasi-TEM characteristic impedance for shielded microstrip lines. Most of the work done on the dispersion characteristics of microstrip line has been numerical in nature, though Schneider [9] obtained an empirical formula for the dispersion characteristics. Fong and Lee [10] and Nefedov and Fialkovskii [11] have derived analytic theories for the dispersion characteristics of wide microstrip line. However, their solutions are only valid when the width of the strip is large compared to wavelength so that coupling between the edges of the strip can be ignored.

In this paper, we use the method of matched asymptotic expansions [12]–[14] to obtain analytic formulas and eigenequations for the effective permittivity of the lowest order mode and the odd and even higher order modes of the microstrip line. The method of matched asymptotic expansions automatically takes into account the coupling between the edge fields of the line. Thus the eigenequations and formulas, which assume simple forms, are valid down to zero frequency. The eigenequations and formulas are asymptotically good when the height-to-width ratio of the line is small and the frequency is finite. The analysis shows that a guided wave does not radiate, and provides better physical insight into the microstrip transmission line problem which have been of continued interest for many years.

II. Method of Solution

An analytical solution to the problem of open microstrip line shown in Fig. 1 can be obtained by the method of matched asymptotic expansions. The method of matched asymptotic expansions has been applied to solve other microstrip problems [7], [15]–[17]. In this method, a small parameter has to be assumed which is d/W in this case. The space around the strip is then divided into three regions; the interior region, the edge region, and the exterior region. The method of matched asymptotic expansions is intricate. The readers are urged to refer to [12], [13], or [17] for the fine points of this method. We shall first illustrate the main ideas by finding the leading order solution in each region when $d/W \to 0$. We assume $\exp(-i\omega t + ik_y y)$ dependence for the guided fields in all regions.

A. The Interior Solution

To find the solution in the interior region when $d/W \to 0$, we emphasize the interior region with the following transformation for the z-coordinate:

$$z = W\delta Z, \qquad \text{where } \delta = d/W. \tag{1}$$

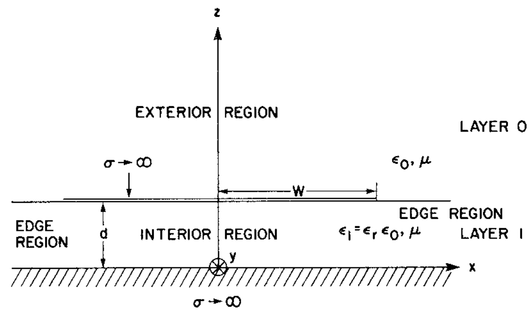

Fig. 1. Geometrical configuration.

Under the transformed coordinate, the leading order solution in the interior region resembles that of a parallel-plate waveguide. Considering only even modes with no Z-variation, it is

$$E_{1z}(x) \sim E_{1z}^{(0)}(x) = E_0 \cos(k_{1x}x) \tag{2}$$

$$H_{1y}(x) \sim H_{1y}^{(0)}(x) = -\frac{ik_{1x}}{\omega\mu} E_0 \sin(k_{1x}x) \tag{3}$$

$$H_{1x}(x) \sim H_{1x}^{(0)}(x) = \frac{k_y}{\omega\mu} E_0 \cos(k_{1x}x) \tag{4}$$

where

$$k_{1x} = \sqrt{k_1^2 - k_y^2}$$

and

$$k_1 = \omega\sqrt{\mu\epsilon_1}\,.$$

The preceding field is seen to satisfy the boundary condition $\hat{n} \times \bar{E}_1 = 0$ at $Z=0$ and 1.

B. The Edge Solution

The edge region is emphasized by the following coordinate transformation:

$$z = W\delta Z \qquad x = W(1+\delta X). \tag{5}$$

Maxwell's equations can then be written in the following form convenient for iteration:

$$\nabla_s \times \bar{e}_{is} = i\omega\mu W\delta \bar{h}_{iy} \tag{6a}$$

$$\nabla_s \times \bar{h}_{is} = -i\omega\epsilon_i W\delta \bar{e}_{iy} \tag{6b}$$

$$\nabla_s e_{iy} = i\omega\mu\delta W\hat{y} \times \bar{h}_{is} + ik_y\delta W\bar{e}_{is} \tag{6c}$$

$$\nabla_s h_{iy} = -i\omega\epsilon_i\delta W\hat{y} \times \bar{e}_{is} + ik_y\delta W\bar{h}_{is} \tag{6d}$$

where the subscript i indicates the solution in layer i of the edge region, the subscript s denotes fields transverse to the y-axis and $\nabla_s = \hat{x}(\partial/\partial X) + \hat{z}(\partial/\partial Z)$. When $\delta \to 0$, the leading order transverse fields are curl-free. The solution is thus a static solution involving semi-infinite half-plane, which can be solved by the Wiener–Hopf technique. The detail of deriving the solution in such a problem is shown in [17] or [18]. Hence, the leading order transverse fields are given by

$$\bar{e}_{is}^{(0)} = -B_1 \nabla_s \Phi_i^{(0)}(\epsilon_r)$$

$$\bar{h}_{is}^{(0)} = -B_2 \nabla_s \psi_i^{(0)}(\epsilon_r = 1). \tag{7}$$

In the preceding, $\Phi_i^{(0)}(\epsilon_r)$ and $\psi_i^{(0)}(\epsilon_r=1)$ are electric and magnetic potentials given in Appendix A.

The leading order $\hat{y}$-components of the fields are of $O(\delta)$ as can be seen from (6). Thus we let $e_{iy}\sim\delta e_{iy}^{(0)}$ and $h_{iy}\sim\delta h_{iy}^{(0)}$. By substituting (7) into (6c–d), we deduce that

$$e_{iy}^{(0)}(X,Z)=iWB_1\cdot\left[\omega\mu\frac{B_2}{B_1}\Phi_i^{(0)}(\epsilon_r=1)-k_y\Phi_i^{(0)}(\epsilon_r)\right] \quad (8)$$

$$h_{iy}^{(0)}(X,Z)=iWB_2\left[\omega\epsilon_i\frac{B_1}{B_2}\psi_i^{(0)}(\epsilon_r)-k_y\psi_i(\epsilon_r=1)\right]+C_1 \quad (9)$$

where B_1, B_2, and C_1 are as yet undetermined constants. In order to have $e_{iy}^{(0)}$ $(Z=1, X<0)=0$, we arrive at $B_2/B_1=k_y/\omega\mu$, since $\Phi_i^{(0)}$ $(Z=1, X<0)=1$. To determine the other unknowns in (9), we attempt an asymptotic matching of (9) to the interior solution. From (4), we obtain the edge expansion of the interior solution $H_{1y}^{(0)}$, viz.,

$$H_{1y}^{(0)}(W(1+\delta X),Z)\sim-\frac{ik_{1x}E_0}{\omega\mu}\sin k_{1x}W-\frac{ik_{1x}^2}{\omega\mu}E_0\cos(k_{1x}W)\delta WX, \qquad \delta\to 0. \quad (10)$$

Furthermore, the interior expansion of the edge solution (9) can be obtained by the asymptotic expansion of the integral (A.4) as is done in [15]–[18]. The result is

$$h_{1y}^{(0)}\left(\frac{x-W}{W\delta},Z\right)\sim iB_2k_y^{-1}k_{1x}^2\delta WX+C_1 + \text{exponentially small terms}, \qquad \delta\to 0. \quad (11)$$

Comparing (10) and (11), the matching condition determines the values of B_2 and C_1. We thus have

$$B_1=-E_0\cos k_{1x}W$$

$$B_2=-\frac{k_y}{\omega\mu}E_0\cos k_{1x}W$$

$$C_1=-\frac{ik_{1x}}{\omega\mu}E_0\sin k_{1x}W \quad (12)$$

which determines the leading order edge solution uniquely.

C. The Exterior Solution

When $\delta\to 0$, the leading order exterior solution has to satisfy the boundary condition $\hat{n}\times\overline{E}_0(x,z=0)=0$, $|x|<W$ and $|x|>W$, and it has to match asymptotically to the edge solution near the edge. The exterior expansion of the edge solution can be deduced by the asymptotic expansions of the integrals [15] in (A.1) and (A.4) for substitution in (8) and (9). The result is

$$e_{0y}\left(\frac{x-W}{W\delta},\frac{z}{W\delta}\right)\sim 0(\delta), \qquad \delta\to 0 \quad (13)$$

and

$$h_{0y}\left(\frac{x-W}{W\delta},\frac{z}{W\delta}\right)\sim-\frac{i\delta W}{\pi\omega\mu}E_0\cos k_{1x}W\cdot\left[k_\rho^2\ln\left(\frac{\left[(x-W)^2+z^2\right]^{1/2}}{W\delta}\right)+\left(k_0^2A-k_y^2(\ln\pi+1)\right)\right]-\frac{ik_{1x}}{\omega\mu}E_0\sin k_{1x}W \quad (14)$$

where

$$A=-2\epsilon_r\sum_{k=2}^{\infty}\left(\frac{1-\epsilon_r}{1+\epsilon_r}\right)^k\ln(k)+\epsilon_r\ln\pi+(\epsilon_r-1)\ln 2+1 \quad (14a)$$

and

$$k_\rho=\sqrt{k_0^2-k_y^2}. \quad (14b)$$

The leading order exterior solution is hence

$$E_{0y}\sim\delta E_{0y}^{(0)}$$
$$H_{0y}\sim\delta H_{0y}^{(0)} \quad (15)$$

where $E_{0y}^{(0)}=0$ and

$$H_{0y}^{(0)}(x,z)=\frac{E_0\cos(k_{1x}W)}{2\omega\mu}\cdot k_\rho^2\left[H_0^{(1)}(k_\rho\rho_1)-H_0^{(1)}(k_\rho\rho_2)\right]W \quad (16)$$

where

$$\rho_1=\sqrt{(x+W)^2+z^2}$$
$$\rho_2=\sqrt{(x-W)^2+z^2}$$

and $H_0^{(1)}(x)$ is the Hankel function of the first kind. Equation (16) resembles the field due to two parallel magnetic line sources. It is also uniquely determined by the boundary and matching conditions.

We can deduce the edge expansion of H_{0y} which is

$$H_{0y}\sim-\frac{i\delta WE_0\cos k_{1x}W}{\pi\omega\mu}k_\rho^2\cdot\left\{\ln\left(\frac{\left[(x-W)^2+z^2\right]^{1/2}}{W}\right)+\ln\frac{k_\rho w}{2}+\gamma-\frac{i\pi}{2}\cdot\left[1-H_0^{(1)}(2k_\rho W)\right]\right\} \quad (17)$$

where γ is the Euler constant. Comparing (17) and (14), we can deduce an eigenequation from the matching condition given asymptotically by

$$k_{1x}\tan(k_{1x}W)\sim\frac{\delta W}{\pi}\left\{k_\rho^2\left[\ln\delta+\ln\frac{k_\rho W}{2}+\gamma-i\frac{\pi}{2}\left(1-H_0^{(1)}(2k_\rho W)\right)\right]-k_0^2A+k_y^2(\ln\pi+1)\right\},\qquad \delta\to 0. \quad (18)$$

Since k_{1x} and k_ρ are functions of k_y, (18) can be solved asymptotically for k_y for different modes. The effective permittivity ratio, $\epsilon_e=k_y^2/k_0^2$, is given approximately by

$$\epsilon_e\sim\epsilon_r-\left(\frac{n\pi}{\tilde{W}}\right)^2+\frac{\delta}{\pi}\left\{\alpha^2\left[\ln\left(\frac{\delta\alpha\tilde{W}}{2}\right)+\gamma+K_0(2\alpha\tilde{W})\right]+A-\tilde{k}_y^2(\ln\pi+1)\right\},\qquad \delta\to 0 \quad (19)$$

where $\tilde{W}=k_0W$, $\tilde{k}_y=\sqrt{\epsilon_r-(n\pi/\tilde{W})^2}$, $\alpha=\sqrt{\tilde{k}_y^2-1}$, $K_0(x)$ is the modified Bessel function of the second kind, and n is the order of the modes on the microstrip line. The preceding is an asymptotic formula for the ϵ_e in the first approximation.

The higher order approximations can be carried out similar to [15] and [17]. We can derive eigenequations for both the even and the odd modes. The details are illustrated in Appendix B. The resulting eigenequations for the even and odd modes are given by

$$\mp\frac{\tilde{k}_{1x}\left[\tan(\tilde{k}_{1x}\tilde{W})\right]^{\pm 1}}{\tilde{W}}\sim\frac{\delta}{\pi}\alpha^2(\ln\delta+H)\pm\left(\frac{\delta}{\pi}\right)^2\cdot\frac{\epsilon_e\epsilon_r-1}{\epsilon_r}2\alpha\tilde{W}K_1(2\alpha\tilde{W})+\left(\frac{\delta}{\pi}\right)^2\cdot\left\{\frac{\alpha\tilde{W}(\epsilon_e\epsilon_r-1)}{\epsilon_r}\left[\frac{1}{2\alpha\tilde{W}}\pm K_1(2\alpha\tilde{W})G\right]\pm\frac{\alpha\tilde{W}}{\epsilon_r}\left[2A+\ln 2-\epsilon_e\epsilon_r(\ln 2\pi^2+2)\right]\cdot K_1(2\alpha\tilde{W})\pm\pi^2\frac{\alpha\tilde{W}}{2}\left(\frac{1-\epsilon_r}{\epsilon_r}\right)\cdot\exp(-2\alpha\tilde{W})\pm I(\epsilon_e)\right\},\qquad \delta\to 0 \quad (20)$$

where $\tilde{W}$ is as defined before, $\alpha=\sqrt{\epsilon_e-1}$, $\tilde{k}_{1x}=\sqrt{\epsilon_r-\epsilon_e}$,

$$G=\ln(\alpha\tilde{W}/2)\pm K_0(2\alpha\tilde{W})+\gamma \quad (20a)$$

$$H=G+\left[A-\epsilon_e(\ln\pi+1)\right]/\alpha^2$$

$$I(\epsilon_e)=\alpha^3\tilde{W}\int_0^{2\alpha\tilde{W}}du\left\{\frac{1-\epsilon_e\epsilon_r}{\alpha^2\epsilon_r}\left[K_1(u)K_1(2\alpha\tilde{W}-u)-K_1(2\alpha\tilde{W})K_1(2\alpha\tilde{W}-u)-\frac{K_1(2\alpha\tilde{W})}{u}\pm K_1^2(2\alpha\tilde{W}-u)\mp\frac{1}{(2\alpha\tilde{W}-u)^2}\right]+\left[K_0(u)\pm K_0(2\alpha\tilde{W}-u)\right]K_0(2\alpha\tilde{W}-u)\right\}. \quad (20b)$$

In the preceding, the upper sign is chosen for the even modes and the lower sign for the odd modes. The eigenequations can be solved approximately giving the effective permittivity ratio to order δ^2

$$\epsilon_e\sim\epsilon_r-\left(\frac{(n+1/4\mp 1/4)\pi}{\tilde{W}}\right)^2+\frac{\delta\alpha^2}{\pi}(\ln\delta+H)+\left(\frac{\alpha\delta\ln\delta}{\pi}\right)^2+\left(\frac{\delta}{\pi}\right)^2\ln\delta\left\{\alpha^2\left[2G\mp\alpha\tilde{W}K_1(2\alpha\tilde{W})+\frac{1}{2}\right]+\left[A-(2\tilde{k}_y^2-1)(\ln\pi+1)\right]\pm\frac{\tilde{k}_y^2\epsilon_r-1}{\epsilon_r}2\alpha\tilde{W}K_1(2\alpha\tilde{W})\right\}+\left(\frac{\delta}{\pi}\right)^2\left\{\alpha^2\left[G\mp\alpha\tilde{W}K_1(2\alpha\tilde{W})-\ln\pi-\frac{1}{2}\right]H+\frac{\alpha\tilde{W}(\tilde{k}_y^2\epsilon_r-1)}{\epsilon_r}\left[\frac{1}{2\alpha\tilde{W}}\pm K_1(2\alpha\tilde{W})G\right]\pm\pi^2\frac{\alpha\tilde{W}}{2}\left(\frac{1-\epsilon_r}{\epsilon_r}\right)\exp(-2\alpha\tilde{W})\pm\frac{\alpha\tilde{W}}{\epsilon_r}\left[2A+\ln 2-\tilde{k}_y^2\epsilon_r\cdot(\ln 2\pi^2+2)\right]K_1(2\alpha\tilde{W})+\frac{\alpha^4W^2}{3}\cdot[\ln\delta+H]^2\pm I\left(\sqrt{\alpha^2-1}\right)\right\},\qquad \delta\to 0\cdot \quad (21)$$

In the preceding $\tilde{W}$, $\tilde{k}_y$, α, and n are as defined for (19). The integral terms I in both (20) and (21) are small and can be neglected for most practical considerations. When $\omega\to 0$, $\tilde{W}\to 0$, (21) reduces to the result of [7], [17] the quasi-TEM approximation for the lowest order even mode.

III. Numerical Results and Discussion

In Fig. 2, the asymptotic eigenequation (20) is solved for the lowest order mode and compared with the results of Jansen [19] which have been accepted as the more reliable of the published numerical data [2], [3]. The set of our curves, labelled as "eigenequation 1", corresponds to neglecting the integral term in (20). We note that the dis-

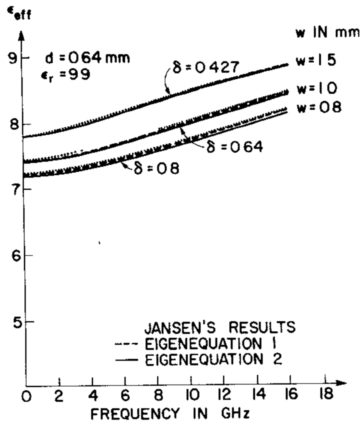

Fig. 2. ϵ_{eff} as a function of frequency for various δ and different approximations to the eigenequation.

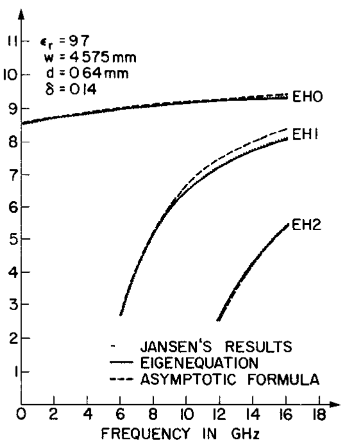

Fig. 3. ϵ_{eff} for various modes and comparison between different approaches.

crepancies that ensue are small, and incidentally, it agrees better with Jansen's numerical results compared to when the full eigenequation (20) is solved for ϵ_e (eigenequation 2). Even though the asymptotic eigenequation (20) is good when $\delta \to 0$, we note that excellent agreement with numerical result is obtained even when $\delta \simeq 0.8$.

In Fig. 3, the asymptotic eigenequation (20) is solved for the effective permittivity of the even and odd modes. The results agree very well with Jansen's. For such a value of δ, the results are not drastically affected even if the integral term in (20) is neglected. We note that the asymptotic formula is excellent for such a δ for the even modes while the asymptotic formula for the odd mode is good when the frequency is lower.

In conclusion, we have derived through the method of matched asymptotic expansions, asymptotic eigenequations for the even and odd modes on a microstrip line. For convenience, we can neglect a complicated term in the equations and the resultant equations, which are simple and do not involve any integration give results which are valid even when $d/W \simeq 0.8$. As opposed to past methods whose eigenequations usually involve integrations, our eigenequation contains modified Bessel functions whose polynomial approximations are well known [20].

From the analysis, we notice that for a guided mode, ϵ_e is always real if ϵ_r is real, implying that a guided wave does not radiate if its phase velocity is slower than that of region 0. This is analogous to the guided modes in a dielectric slab. However, equation (20) does not preclude the possibility of complex roots for real ϵ_r analogous to the leaky waves in a lossless dielectric slab. Also when $\delta \to 0$, the leading order ϵ_e is that of a rectangular waveguide with vertical magnetic wall. The first order correction is of $O(\delta \ln \delta)$ which vanishes slowly when $\delta \to 0$. This indicates the importance of the fringing field whose effect lingers on when $\delta \to 0$. From our exterior solution, we note that the exterior fields are describable by cylindrical waves. When the wave is guided, the cylindrical waves are evanescent in nature representable by modified Bessel functions. This explains the presence of modified Bessel functions in the eigenequations. The decay of the evanescent wave from the microstrip line is proportional to the frequency and the contrast in the phase velocity of the guided wave and a wave in medium zero, in other words, the decay rate $\alpha \sim \omega\sqrt{\mu(\epsilon_e - 1)\epsilon_0}$. So if the frequency is high, and/or $\epsilon_e - 1$ is large, the shielding of the microstrip line is unnecessary since the field is localized around the microstrip line. We can account for the dielectric loss in our approach by letting ϵ_r be complex, since in our analysis, ϵ_r is not restricted to be pure real. In our analysis, we have approximated our edge solution with a two term quasistatic approximation. This is legitimate as long as the frequency is not too high such that the height of the microstrip line is not comparable to the wavelength. Our formula is asymptotic in the sense that for ω, W, and ϵ_r fixed, the result is asymptotically good when $d \to 0$.

Appendix A—Potentials for Edge Solutions

The potential $\phi_i^{(0)}(\epsilon_r)$ and $\psi_i^{(0)}(\epsilon_r)$ of (7)–(9) are given as

$$\phi_0^{(0)}(\epsilon_r) = \frac{i}{2\pi}\int_{-\infty}^{\infty} \frac{G_-(\lambda,\epsilon_r)}{\lambda G_-(0,\epsilon_r)} i \cdot \exp\left[-\alpha(Z-1) + i\lambda X\right] d\lambda \tag{A.1}$$

$$\phi_1^{(0)}(\epsilon_r) = \frac{i}{2\pi}\int_{-\infty}^{\infty} \frac{G_-(\lambda,\epsilon_r)}{\lambda G_-(0,\epsilon_r)} \cdot \left[\frac{\exp[\alpha(Z-1)] - \exp[-2\alpha - \alpha(Z-1)]}{1 - \exp - 2\alpha}\right] \cdot \exp[i\lambda X] d\lambda \tag{A.2}$$

$$\psi_0^{(0)}(\epsilon_r) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{G_-(\lambda,\epsilon_r)}{\lambda^2 G_-(0,\epsilon_r)} \cdot \left[\alpha \exp[i\lambda X - \alpha(Z-1)] + 1\right] d\lambda \tag{A.3}$$

$$\psi_1^{(0)}(\epsilon_r) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{G_-(\lambda,\epsilon_r)}{\lambda^2 G_-(0,\epsilon_r)} \cdot \left[\alpha \frac{\exp[\alpha(Z-1)] + \exp[-2\alpha - \alpha(Z-1)]}{1 - \exp - 2\alpha} \cdot \exp[i\lambda X] - 1\right] d\lambda \tag{A.4}$$

where $\alpha=\lim_{b\to 0}\sqrt{\lambda^2-b^2}$, and $G_-(\lambda,\epsilon_r)$ is described in [17]–[18]. We note that

$$\frac{\partial\psi}{\partial X}=\frac{\partial\phi}{\partial Z}$$
$$\frac{\partial\partial}{\partial Z}=-\frac{\partial\phi}{\partial X}. \tag{A.5}$$

Appendix B—Higher Order Approximation

The forms of higher order solutions in the exterior region can be deduced by observing the higher order exterior expansion of the edge solutions which are derived to be [15], [17]

$$\begin{aligned}h_{0y}\left(\frac{x-W}{W\delta},\frac{z}{W\delta}\right) &\sim -i\delta W\frac{E_0\cos(k_{lx}W)}{\omega\mu\pi}\Big\{k_0^2A-k_y^2(\ln\pi+1)\\&+k_\rho^2\ln(\rho_2/\delta W)+\frac{\delta\ln\delta}{\pi\epsilon_r}\\&\cdot\left(k_0^2-k_y^2\epsilon_r\right)\frac{(x-W)W}{\rho_2^2}\\&-\frac{\delta W}{\pi\epsilon_r}\left(k_0^2-k_y^2\epsilon_r\right)\\&\cdot\left[\frac{z}{\rho_2^2}\tan^{-1}\frac{z}{x-W}+\frac{x-W}{\rho_2^2}\right.\\&\left.\cdot\ln(\rho_2/W)\right]-\frac{\delta W}{\pi\epsilon_r}\\&\cdot\Big[\left(k_0^2A-k_y^2\epsilon_r(\ln\pi+1)\right)\\&\cdot\frac{x-W}{\rho_2^2}+k_0^2\pi(\epsilon_r-1)\frac{z}{\rho_2^2}\Big]\Big\}\\&-i\frac{k_{lx}}{\omega\mu}E_0\sin k_{lx}W,\qquad \delta\to 0\end{aligned} \tag{B.1}$$

$$\begin{aligned}e_{0y}\left(\frac{x-W}{W\delta},\frac{z}{W\delta}\right) &\sim i\delta W\frac{k_yE_0\cos(k_{lx}W)}{\pi}\Big\{\frac{\delta\ln\delta}{\pi\epsilon_r}(\epsilon_r-1)\frac{zW}{\rho_2^2}+\frac{\delta W}{\pi\epsilon_r}\\&\cdot(\epsilon_r-1)\left[\frac{x-W}{\rho_2^2}\tan^{-1}\frac{z}{x-W}\right.\\&\left.-\frac{z}{\rho_2^2}\ln\frac{\rho_2}{W}-\frac{\pi(x-W)}{\rho_2^2}\right]\\&+\frac{\delta W}{\pi\epsilon_r}\left[A-\epsilon_r(\ln\pi+1)\right]\frac{z}{\rho_2^2}\Big\}\end{aligned} \tag{B.2}$$

where A and ρ_2 are defined previously. This suggests that the exterior solutions are of the form

$$H_{0y}\sim\delta H_{0y}^{(0)}+\delta^2\ln\delta H_{0y}^{(1)}+\delta^2H_{0y}^{(2)} \tag{B.3}$$

and

$$E_{0y}\sim\delta^2\ln\delta E_{0y}^{(1)}+\delta^2E_{0y}^{(2)}. \tag{B.4}$$

The boundary conditions for the fields can be expressed in terms of the y-components of the fields since they uniquely determine the other components through the equation

$$\bar{E}_{ts}=\frac{i}{k_i^2-k_y^2}\left(k_y\nabla_sE_{ty}+\omega\mu\nabla_s\times\bar{H}_{ty}\right). \tag{B.5}$$

For example, the vanishing of the tangential $\bar{E}_t$-field in layer-1 at $z=0$ implies that

$$E_{1y}(z=0)=0$$
$$\frac{\partial}{\partial z}H_{1y}(z=0)=0. \tag{B.6}$$

The continuity of tangential $\bar{E}$ field at $z=d$ is the same as

$$E_{0y}(z=d)=E_{1y}(z=d) \tag{B.7a}$$

$$\begin{aligned}k_\rho^2\Big[k_y\frac{\partial}{\partial x}E_{1y}(z=d)&-\omega\mu\frac{\partial}{\partial z}H_{1y}(z=d)\Big]=k_{1\rho}^2\Big[k_y\frac{\partial}{\partial x}E_{0y}(z=d)\\&-\omega\mu\frac{\partial}{\partial z}H_{0y}(z=d)\Big]\end{aligned} \tag{B.7b}$$

where k_ρ is defined in (14b) and $k_{1\rho}=\sqrt{k_1^2-k_y^2}$. The continuity of tangential $\bar{H}$ field at $z=d$ for $|x|>W$, after using an equation dual to (B.5), implies that

$$H_{0y}(z=d)=H_{1y}(z=d),\qquad |x|>W \tag{B.8a}$$

$$\begin{aligned}k_\rho^2\Big[k_y\frac{\partial}{\partial x}H_{1y}(z=d)+\omega\epsilon_0\epsilon_r\frac{\partial}{\partial z}E_{1y}(z=d)\Big]\\=k_{1\rho}^2\Big[k_y\frac{\partial}{\partial x}H_{0y}(z=d)+\omega\epsilon_0\frac{\partial}{\partial z}E_{0y}(z=d)\Big],\\|x|>W.\end{aligned} \tag{B.8b}$$

By Taylor expanding (B.7) and (B.8) about $z=0$, we substitute in the perturbation series suggested by (B.3) and (B.4) for fields in layer 0 and 1 (except we assume that $E_{1y}^{(0)}\neq 0$). By matching terms of the same order, we can show from (B.7) that

$$\frac{\partial}{\partial z}H_{0y}^{(0)}(z=0)=0$$
$$\frac{\partial}{\partial z}H_{0y}^{(1)}(z=0)=0,\qquad \text{all } x \tag{B.9a}$$

and

$$\frac{\partial}{\partial z}H_{0y}^{(2)}(z=0)=\frac{k_yW(k_1^2-k_0^2)}{\omega\mu k_{1\rho}^2}\frac{\partial^2}{\partial x\partial z}\cdot E_{1y}^{(0)}(z=0)-W\frac{\partial^2}{\partial z^2}\cdot\left[H_{0y}^{(0)}(z=0)-\frac{k_{0\rho}^2}{k_{1\rho}^2}H_{1y}^{(0)}(z=0)\right],\quad \text{all } x. \quad (\text{B.9b})$$

By making use of the result of (B.8) and Helmholtz wave equation, we conclude that (B.9b) simplifies to

$$\frac{\partial}{\partial z}H_{0y}^{(2)}(x,z=0)=-\frac{k_0^2W}{k_\rho^2}\left(\frac{1-\epsilon_r}{\epsilon_r}\right)\cdot\frac{\partial^2}{\partial x^2}H_{0y}^{(0)}(x,z=0),\qquad |x|>W. \quad (\text{B.9c})$$

For $|x|<W$, we can show from (B.7a) that $(\partial/\partial z)E_{1y}^{(0)}(z=0)=0$. Hence, the boundary condition for (B.9b) is

$$\frac{\partial}{\partial z}H_{0y}^{(2)}(x,z=0)=W\frac{\partial^2}{\partial x^2}H_{0y}^{(0)}(x,z=0)+k_\rho^2WH_{0y}^{(0)}(x,z=0),\qquad |x|<W. \quad (\text{B.9d})$$

Similarly, from (B.7a), (B.8), and Maxwell equations, we have

$$E_{0y}^{(0)}(z=0)=0$$
$$E_{0y}^{(1)}(z=0)=0,\qquad \text{all } x \quad (\text{B.10a})$$

and

$$E_{0y}^{(2)}(z=0)=\begin{cases} i\omega\mu W\left(\dfrac{1-\epsilon_r}{\epsilon_r}\right)H_{0x}^{(0)}(x,z=0), & |x|>W\\ 0, & |x|<W.\end{cases} \quad (\text{B.10b})$$

It can be shown that the $\hat{y}$-components of the first order fields that satisfy the boundary conditions (B.9), (B.10), and the matching conditions provided by (B.1) and (B.2) are

$$H_{0y}^{(1)}=\frac{k_\rho W^2E_0\cos(k_{1x}W)}{2\omega\mu\pi\epsilon_r}(k_0^2-k_y^2\epsilon_r)\cdot\left[\frac{x+W}{\rho_1}H_1^{(1)}(k_\rho\rho_1)+\frac{x-W}{\rho_2}H_1^{(1)}(k_\rho\rho_2)\right] \quad (\text{B.11})$$

$$E_{0y}^{(1)}=-\frac{k_yk_\rho W^2E_0\cos(k_{1x}W)}{2\pi\epsilon_r}(\epsilon_r-1)\cdot\left(\frac{z}{\rho_1}H_1^{(1)}(k_\rho\rho_1)+\frac{z}{\rho_2}H_1^{(1)}(k_\rho\rho_2)\right). \quad (\text{B.12})$$

The preceding resembles fields due to two parallel lines of magnetic and electric dipoles. The solutions of the second order fields that satisfy boundary conditions (B.9) and (B.10) and the matching condition provided by (B.1) and (B.2) are derived as

$$\begin{aligned} H_{0y}^{(2)}=\frac{k_\rho^2W^2E_0\cos(k_{1x}W)}{4i\omega\mu}\cdot\Bigg\{&k_\rho^2\int_{-W}^{W}db\left[H_0^{(1)}(k_\rho(W+b))-H_0^{(1)}(k_\rho(W-b))\right]H_0^{(1)}(k_\rho\rho')+\frac{k_0^2-k_y^2\epsilon_r}{\epsilon_r}\\ &\cdot\left[\int_{-W}^{W}db\left[H_1^{(1)}(k_\rho(W+b))+H_1^{(1)}(k_\rho(W-b))\right]\cdot\frac{x-b}{\rho'}H_1^{(1)}(k_\rho\rho')+\frac{2i}{\pi k_\rho}\int_{-W}^{W}db\right.\\ &\left.\cdot\left[\frac{x+W}{(W+b)\rho_1}H_1^{(1)}(k_\rho\rho_1)+\frac{x-W}{(W-b)\rho_2}H_1^{(1)}(k_\rho\rho_2)\right]\right]\\ &+\frac{4ik_0^2}{\pi k_\rho^2}\left(\frac{1-\epsilon_r}{\epsilon_r}\right)\int_{-\infty}^{\infty}dk_x\frac{k_x^2}{k_z^2}\cdot\sin(k_xW)\exp(ik_xx+ik_zz)\\ &-\frac{2i}{\pi k_\rho\epsilon_r}\left[k_0^2(A+\ln 2)-k_y^2\epsilon_r(\ln 2\pi+1)\right]\\ &\cdot\left[\frac{x+W}{\rho_1}H_1^{(1)}(k_\rho\rho_1)+\frac{x-W}{\rho_2}H_1^{(1)}(k_\rho\rho_2)\right]\Bigg\} \end{aligned} \quad (\text{B.13})$$

$$\begin{aligned} E_{0y}^{(2)}=\frac{k_\rho^2W^2k_yE_0\cos(k_{1x}W)}{2\epsilon_r}\Bigg\{&\frac{i}{2}(\epsilon_r-1)\int_{-W}^{W}db\cdot\left[H_1^{(1)}(k_\rho(W+b))+H_1^{(1)}(k_\rho(W-b))\right]\frac{z}{\rho'}H_1^{(1)}\\ &\cdot(k_\rho\rho')-\frac{(\epsilon_r-1)}{\pi k_\rho}\int_{-W}^{W}db\left[\frac{1}{W+b}\frac{z}{\rho_1}H_1^{(1)}(k_\rho\rho_1)+\frac{1}{W-b}\frac{z}{\rho_2}H_1^{(1)}(k_\rho\rho_2)\right]-\frac{(\epsilon_r-1)}{k_\rho}\\ &\cdot\left[\frac{x+W}{\rho_1}H_1^{(1)}(k_\rho\rho_1)-\frac{x-W}{\rho_2}H_1^{(1)}(k_\rho\rho_2)\right]\\ &-\frac{1}{\pi k_\rho}\left[A-\epsilon_r(\ln\pi+1)-(\epsilon_r-1)\ln 2\right]\\ &\cdot\left[\frac{z}{\rho_1}H_1^{(1)}(k_\rho\rho_1)+\frac{z}{\rho_2}H_1^{(1)}(k_\rho\rho_2)\right]\Bigg\} \end{aligned} \quad (\text{B.14})$$

where $\rho'=\sqrt{(x-b)^2+z^2}$, where b is the integration variable in (B.13) and (B.14), and $k_z=\sqrt{k_0^2-k_x^2}$. The last integrals in (B.13) and (B.14) are needed to cancel the infinite terms in the preceding integrals and the last terms are eigensolutions needed to satisfy the matching conditions. We note that the zeroth- and first-order fields are due to the sources at the edges of the microstrip line while the second-order fields have contributions from sources distributed over the strip as well. With the higher order exterior solutions derived, we can find the higher order edge solutions from the edge expansions of the exterior solutions, which are

$$H_{0y}(W+\delta WX,\delta WZ)\sim\delta H_{0y}^{(0)}+\delta^2\ln\delta H_{0y}^{(0)}+\delta^2 H_{0y}^{(2)}$$

$$\sim-\frac{i\delta WE_0\cos k_{1x}W}{\pi\omega\mu}\Bigg\{k_\rho^2\Bigg[\ln(X^2+Z^2)^{1/2}+\ln\frac{\delta k_\rho W}{2}$$

$$+\frac{i\pi}{2}\left(H_0^{(1)}(2k_\rho W)-1\right)+\gamma\Bigg]+\delta\ln\delta$$

$$\cdot\frac{ik_\rho W\left(k_0^2-k_y^2\epsilon_r\right)}{2\epsilon_r}H_1^{(1)}(2k_\rho W)+\delta\frac{\pi k_\rho^4 W}{4}\int_{-W}^{W}db$$

$$\cdot\left[H_0^{(1)}(k_\rho(W+b))-H_0^{(1)}(k_\rho(W-b))\right]$$

$$\cdot H_0^{(1)}(k_\rho(W-b))+\frac{\delta\pi k_\rho^2 W}{4\epsilon_r}\left(k_0^2-k_y^2\epsilon_r\right)$$

$$\cdot\Bigg[\int_{-W}^{W}db\Bigg[H_1^{(1)}(k_\rho(W+b))H_1^{(1)}(k_\rho(W-b))$$

$$-H_1^{(1)}(2k_\rho W)H_1^{(1)}(k_\rho(W-b))$$

$$+\frac{2i}{\pi k_\rho}\frac{H_1^{(1)}(2k_\rho W)}{W+b}+H_1^{(1)}(k_\rho(W-b))^2$$

$$+\frac{4}{\pi^2 k_\rho^2(W-b)^2}\Bigg]-\frac{H_1^{(1)}(2k_\rho W)}{k_\rho}\Bigg[H_0^{(1)}(2k_\rho W)-1$$

$$-\frac{2i}{\pi}\left(\ln\frac{\delta k_\rho W}{2}+\gamma\right)\Bigg]+\frac{2}{\pi^2 k_\rho^2 W}\Bigg]$$

$$+\delta\frac{\pi i k_\rho k_0^2 W}{2}\left(\frac{\epsilon_r-1}{\epsilon_r}\right)\exp(2ik_\rho W)$$

$$-\frac{\delta W i k_\rho}{2\epsilon_r}\left[k_0^2(A+\ln 2)-k_y^2\epsilon_r(\ln 2\pi+1)\right]H_1^{(1)}(2k_\rho W)$$

$$-\frac{k_0^2-k_y^2\epsilon_r}{\pi\epsilon_r}\left[\frac{X\ln(X^2+Z^2)^{1/2}+Z\tan^{-1}\frac{Z}{X}}{X^2+Z^2}\right]$$

$$+k_0^2\left(\frac{1-\epsilon_r}{\epsilon_r}\right)\frac{Z}{X^2+Z^2}-\frac{1}{\pi\epsilon_r}\left[k_0^2 A-k_y^2\epsilon_r\right.$$

$$\left.\cdot(\ln\pi+1)\right]\frac{X}{X^2+Z^2}\delta X\frac{\pi i k_\rho^3}{2}WH_1^{(1)}(2k_\rho W)$$

$$+\delta W\frac{ik_\rho}{2}H_1^{(1)}(2k_\rho W)\frac{k_0^2-k_y^2\epsilon_r}{\epsilon_r}$$

$$\cdot\ln(X^2+Z^2)^{1/2}\Bigg\},\qquad\delta\to 0\tag{B.15}$$

and

$$E_{0y}(W+\delta WX,\delta WZ)\sim\delta^2\ln\delta E_{0y}^{(1)}+\delta^2 E_{0y}^{(2)}$$

$$\sim\frac{i\delta k_y W E_0\cos k_{1x}W}{\pi^2\epsilon_r}$$

$$\cdot\Bigg\{(\epsilon_r-1)\frac{X\tan^{-1}\frac{Z}{X}-Z\ln(X^2+Z^2)^{1/2}}{X^2+Z^2}$$

$$-(\epsilon_r-1)\frac{\pi X}{X^2+Z^2}$$

$$+\left[A-\epsilon_r(\ln\pi+1)\right]\frac{Z}{X^2+Z^2}$$

$$-\delta W\frac{\pi i k_\rho}{2}(\epsilon_r-1)H_1^{(1)}(2k_\rho W)$$

$$\cdot\left(\tan^{-1}\frac{Z}{X}-\pi\right)\Bigg\},\qquad\delta\to 0.\tag{B.16}$$

From (B.15) and (B.16), we deduce that the edge solutions are of the form

$$h_{iy}=\delta h_{iy}^{(0)}+\delta^2 h_{iy}^{(1)}\tag{B.17}$$

$$e_{iy}=\delta e_{iy}^{(0)}+\delta^2 e_{iy}^{(1)}.\tag{B.18}$$

The first-order edge solutions for the y-component of the fields that satisfy (6), the boundary condition and the matching conditions provided by (B.15) and (B.16) are

$$e_{iy}^{(1)}=k_\rho k_y W^2 E_0\cos(k_{1x}W)H_1^{(1)}(2k_\rho W)\cdot\left[\Phi_i^{(1)}(\epsilon_r=1)-\Phi_i^{(1)}(\epsilon_r)\right]\tag{B.19}$$

$$h_{iy}^{(1)}=\frac{k_\rho W^2}{\omega\mu}E_0\cos(k_{1x}W)H_1^{(1)}(2k_\rho W)\cdot\left[k_i^2\psi_i^{(1)}(\epsilon_r)-k_y^2\psi_i^{(1)}(\epsilon_r=1)\right]\tag{B.20}$$

where

$$\Phi_0^{(1)}(\epsilon_r)=\frac{1}{2\epsilon_r}\Phi_0^{(0)}(\epsilon_r)+\frac{1}{2}\left(1-\frac{1}{\epsilon_r}-Z\right)$$

$$\Phi_1^{(1)}(\epsilon_r)=\frac{1}{2\epsilon_r}\Phi_1^{(0)}(\epsilon_r)-\frac{Z}{2\epsilon_r}$$

$$\psi_0^{(1)}(\epsilon_r)=\frac{1}{2\epsilon_r}\psi_0^{0}(\epsilon_r)-\frac{X}{2}$$

$$\psi_1^{(1)}(\epsilon_r)=\frac{1}{2\epsilon_r}\psi_1^{(0)}(\epsilon_r)-\frac{X}{2\epsilon_r}.\tag{B.21}$$

To find the higher order interior solutions, we expand the edge solutions (B.17) and (B.18) in the interior region. It is found that only exponentially small higher order terms are induced by the edge solutions. Therefore, the interior solution is as given by (2)–(4). With the edge solution for the h_{iy} component given by (B.17), we can derive its exterior expansion leading to

$$\begin{aligned} &h_{0y}(X,Z)\\ &\sim \delta h_{0y}^{(0)}+\delta^2 h_{0y}^{(1)} \sim -i\delta W \frac{E_0\cos(k_{1x}W)}{\omega\mu\pi}\\ &\cdot\Bigg\{k_0^2A-k_y^2(\ln\pi+1)+k_\rho^2\ln(X^2+Z^2)^{1/2}-\frac{k_0^2-k_y^2\epsilon_r}{\pi\epsilon_r}\\ &\cdot\left[\frac{Z}{X^2+Z^2}\tan^{-1}\frac{Z}{X}+\frac{X}{X^2+Z^2}\ln(X^2+Z^2)^{1/2}\right]\\ &-\frac{1}{\pi^2\epsilon_r}\Bigg[\left[k_0^2A-k_y^2\epsilon_r(\ln\pi+1)\right]\frac{X}{X^2+Z^2}\\ &+k_0^2\pi(\epsilon_r-1)\frac{Z}{X^2+Z^2}\Bigg]+\delta W\frac{ik_\rho}{2}H_1^{(1)}(2k_\rho W)\\ &\cdot\Bigg[\frac{k_0^2-k_y^2\epsilon_r}{\epsilon_r}\ln(X^2+Z^2)^{1/2}\\ &+\frac{k_0^2A}{\epsilon_r}-k_y^2(\ln\pi+1)-k_\rho^2\pi X\Bigg]\Bigg\}\\ &-\frac{ik_{1x}}{\omega\mu}E_0\sin(k_{1x}W), \qquad (X^2+Z^2)^{1/2}\to\infty. \qquad \text{(B.22)} \end{aligned}$$

Comparing (B.22) and (B.15), and invoking the matching condition, we can deduce an asymptotic eigenequation. We can perform a similar analysis for the odd modes. The resultant asymptotic eigenequations are as given in (20).

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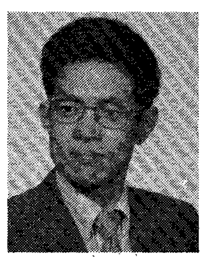

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